

Fast methods to track grain coagulation and ionization

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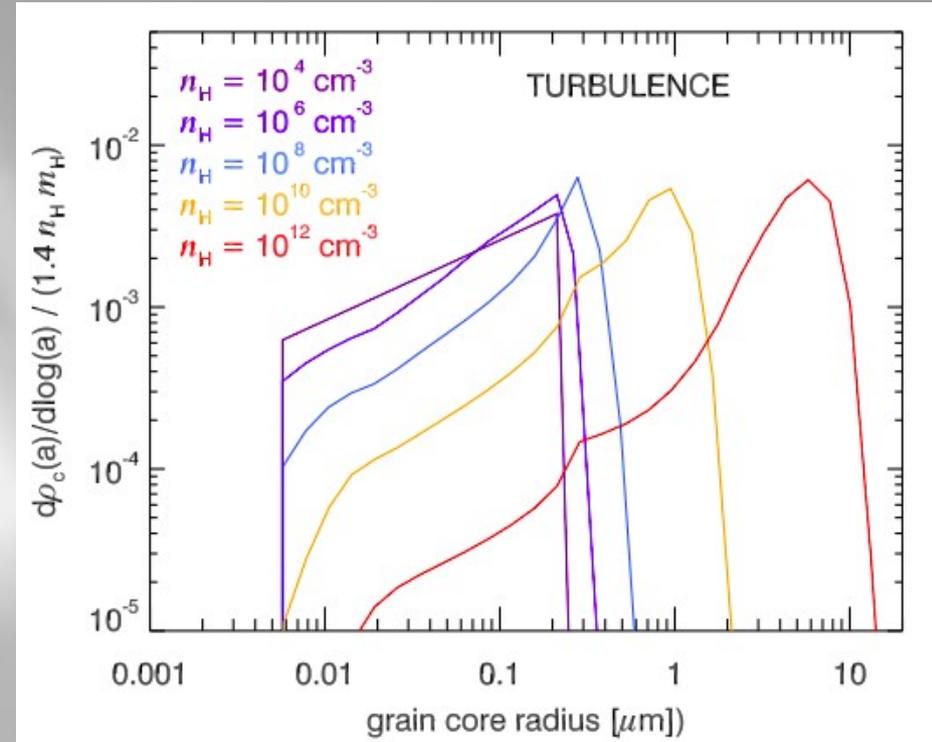
Dust grains represent only 1% of the gas mass in the ISM.

BUT they play a major role in star formation

- Planet formation
- Cooling
- Radiative hydrodynamics
- Radiative transfer
- Non-ideal MHD
- Chemistry
- ...

Size matters

Grains coagulate during star formation



Guillet et al. (2020)

Coagulation algorithms are expensive in hydro simulations

Smoluchowski equation
(Smoluchowski 1916, Mizuno 1988)

$$\begin{aligned} \frac{d\rho(m, t)}{dt} = & - \int_0^\infty mK(m, m')n(m, t)n(m', t)dm' \\ & + \frac{1}{2} \int_0^m mK(m - m', m')n(m - m', t)n(m', t)dm' \\ & + \frac{\rho(m, t)}{n_H} \frac{dn_H}{dt}, \end{aligned}$$

New form with the kernel
of Ormel & Cuzzi (2008)

$$\begin{aligned} \frac{dX(a, \chi)}{d\chi} &= C_2 I(a, X, \chi). \\ \chi &= \int_0^t n_H^{\frac{3}{4}} T^{-\frac{1}{4}} dt. \end{aligned}$$

Grain quantity

No environment
dependency

Only dependency
variable
= summary of
grain history

Marchand et al (2021)

A coagulating size-distribution of grain follows a unique path determined by χ .
Different environments only modify the time-speed of the evolution.

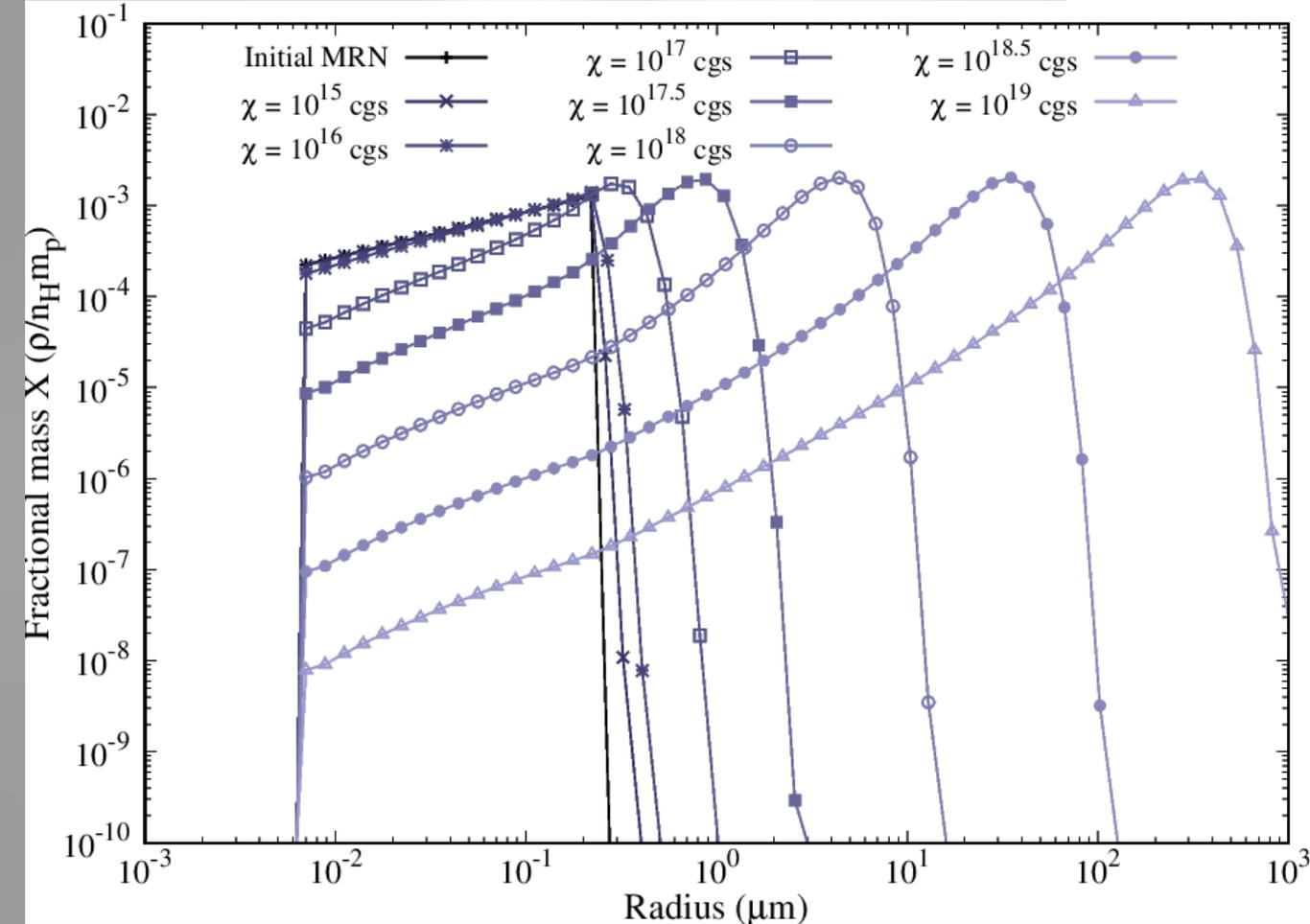
A grain size-distribution is uniquely determined by the initial distribution and χ .

Fast method to track coagulation

Works for every kernel in the form $K = f(n_H, T)g(a)$

Example: MRN distribution

$$\chi = \int_0^t n_H^{4.5} T^{-1/4} dt.$$



How to use this method ?

Before the simulation

- Tabulate the distributions for various χ .

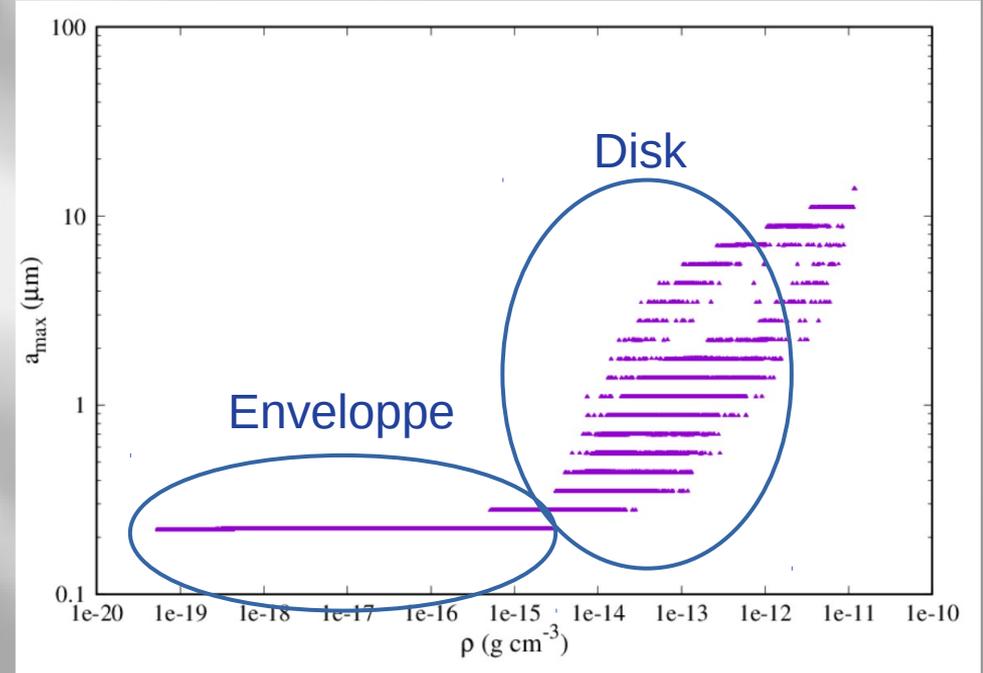
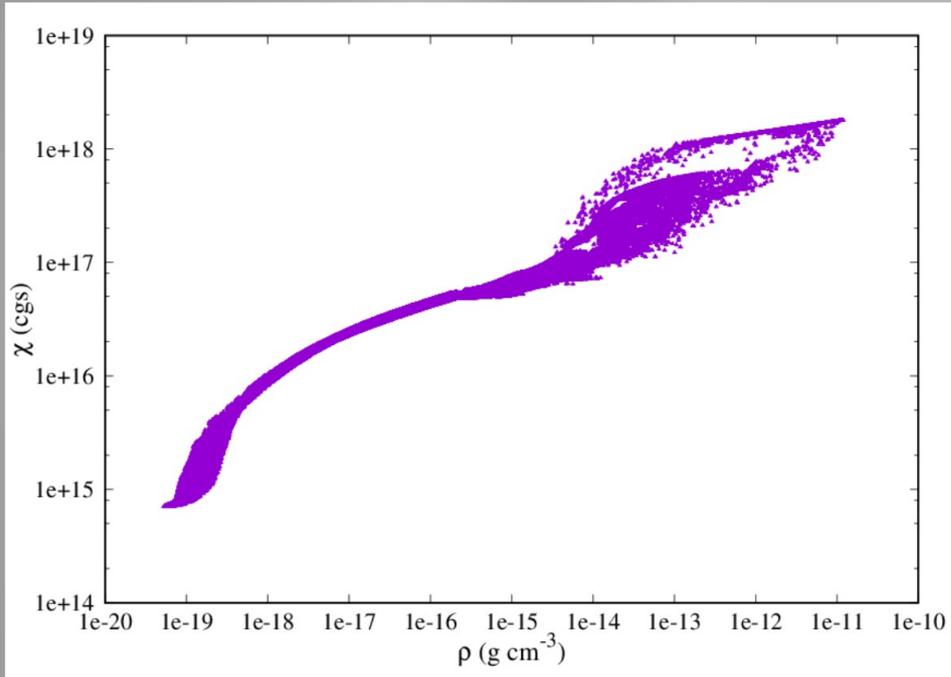
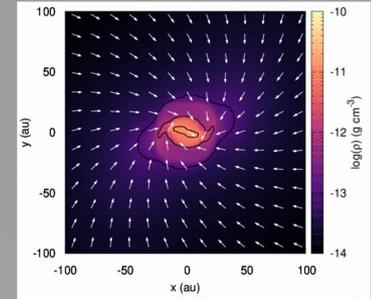
During the simulation

- Calculate χ ,
- Read/interpolate the table to get the distribution,
- Do physics with grains.

In RAMSES: passive scalar

$$\frac{\partial \rho \chi}{\partial t} + \nabla \cdot (\rho \chi \mathbf{u}) = \rho n_{\text{H}}^{\frac{3}{4}} T^{-\frac{1}{4}}$$

χ and mode of the distribution for a protostellar collapse



Marchand et al (in prep)

A fast way to calculate the ionization (Marchand et al, 2021)

Based on Draine & Sutin (1987):

$$Z_k = \psi \tau_k + \frac{1 - \epsilon^2 \Theta^2}{1 + \epsilon \Theta \alpha_k + \epsilon^2 \Theta^2},$$

Grain charge

$$\langle \tilde{J}(\tau_k) \rangle = (1 - \psi) + \frac{\frac{2}{\tau_k} [\epsilon^2 \Theta^2 + \epsilon \Theta]}{1 + \epsilon \Theta \alpha_k + \epsilon^2 \Theta^2},$$

Grain polarization

$$\epsilon = \frac{1 - \psi}{\Theta e^\psi},$$

Ion/electron flux on grains

$$n_i = - \frac{1}{1 - \epsilon} \sum_k n_k Z_k,$$

Charge neutrality

$$f(\psi) = \frac{\langle \sigma v \rangle_{ie} \epsilon n_i^2}{\zeta n_H} + \frac{n_i v_i}{\zeta n_H} \sum_k n_k \pi a_k^2 \langle \tilde{J}(\tau_k) \rangle - 1 = 0,$$

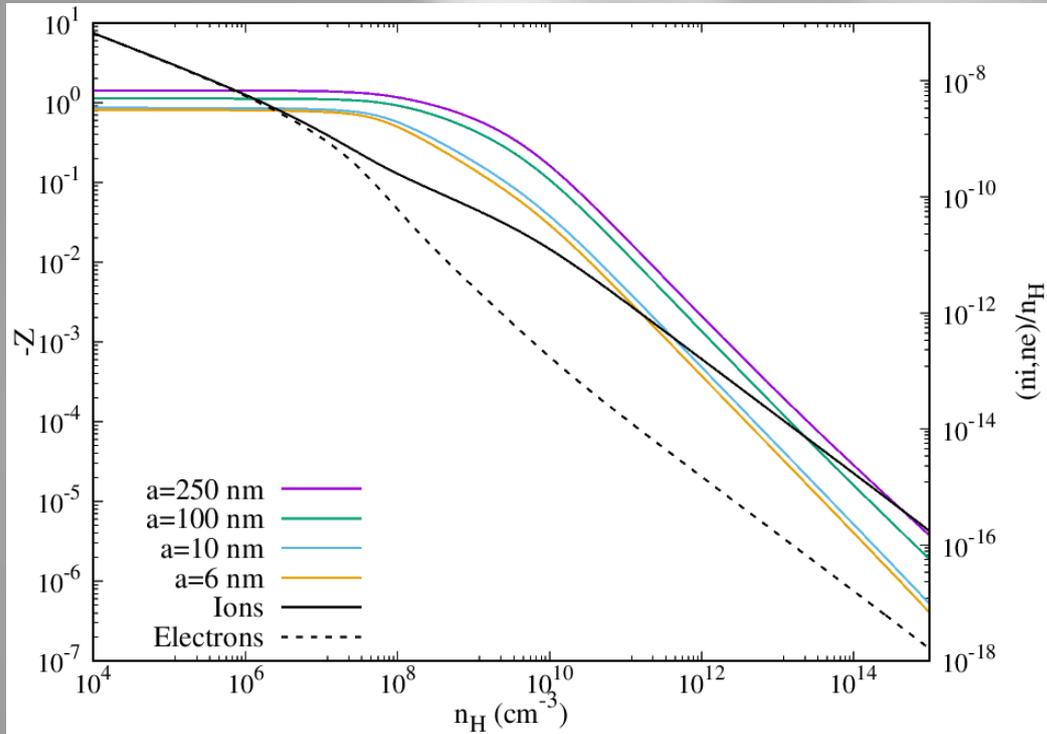
Ionization equilibrium

1 equation to solve for Ψ

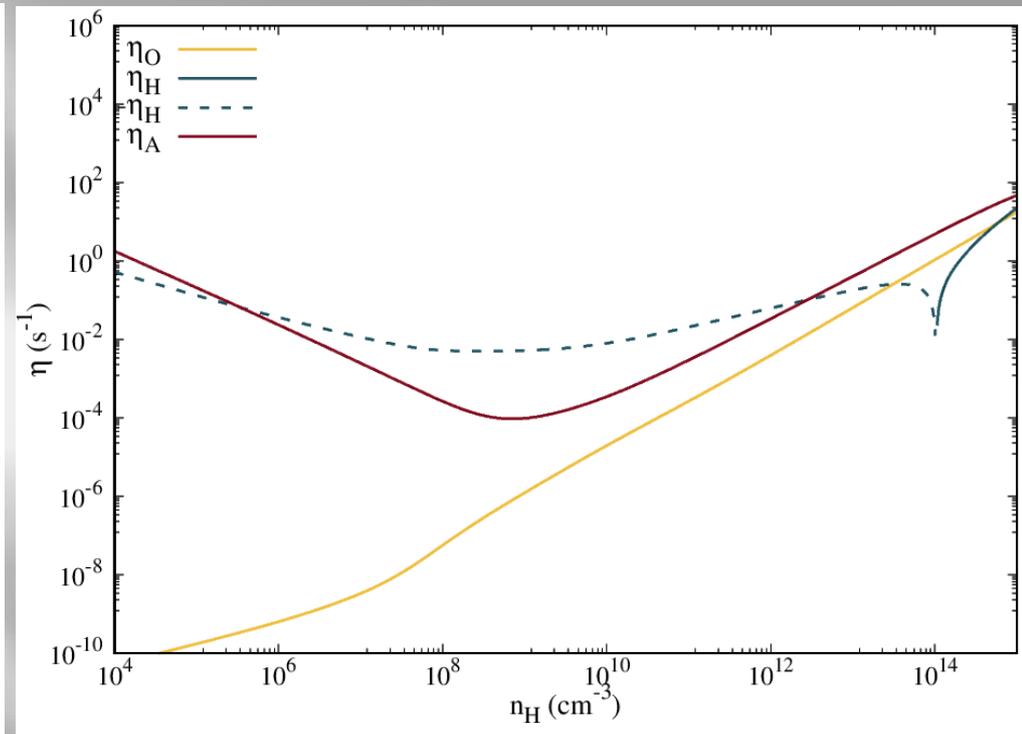
- n_i
- n_e
- Z_k

~ 4 iterations of Newton-Raphson

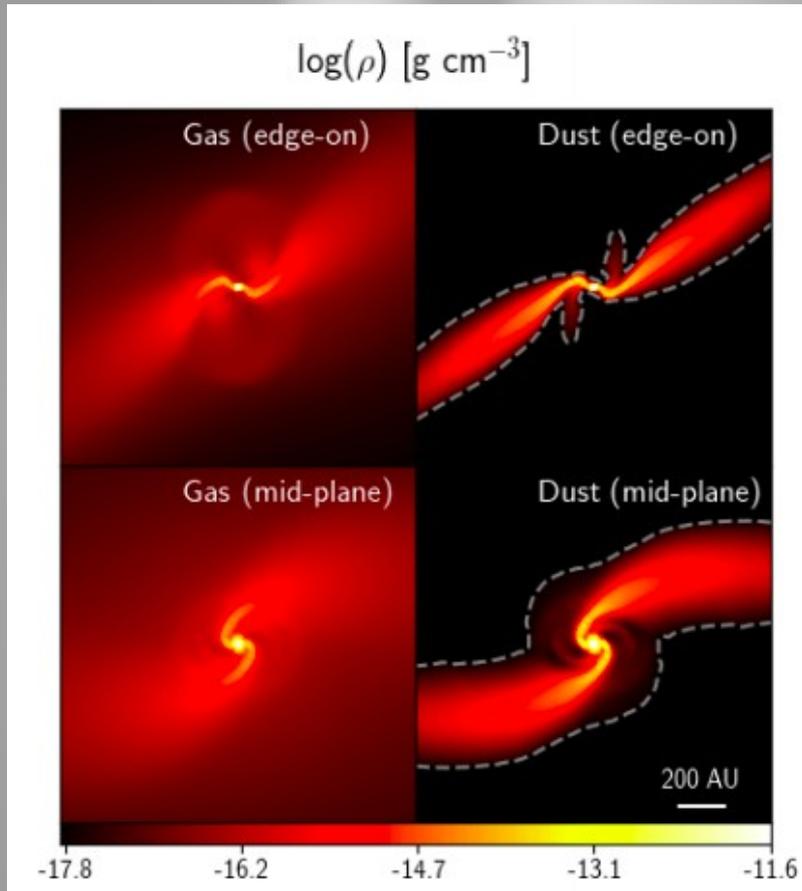
Grain charge / ions-electrons abundances



Non-ideal MHD resistivities

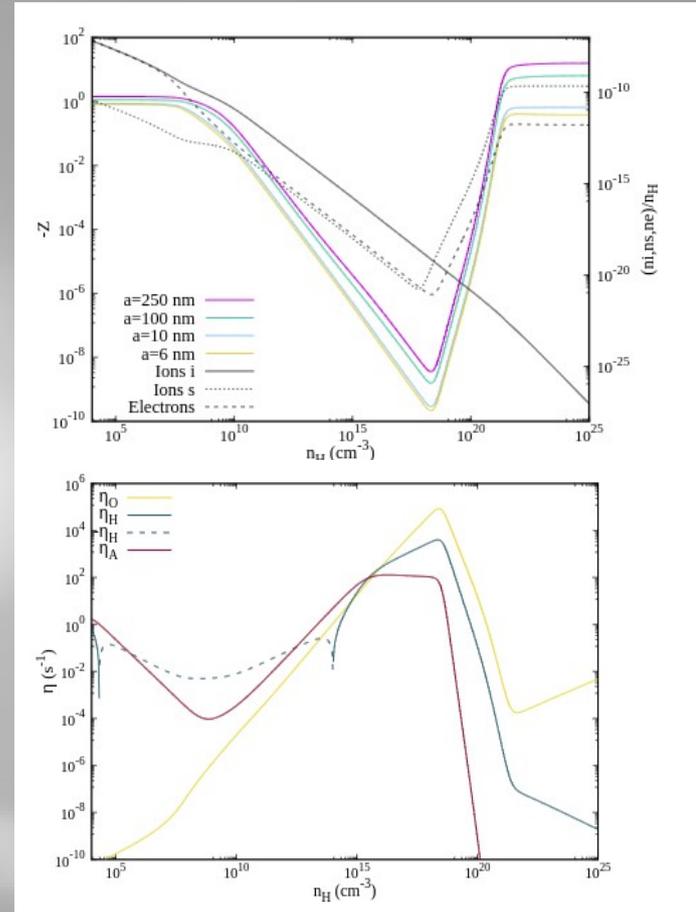


Account for varying dust-to-gas ratio



Lebreuilly et al. (2020)

Include the thermal ionization



Marchand et al. (in prep)

Fast method to track grain coagulation

- Follow the dust growth cheaply,
- Dust grains grow to $\sim 10 \mu\text{m}$ as they enter the protoplanetary disk.

Fast method to calculate the ionization

- Calculate cheaply and self-consistently the non-ideal MHD resistivities.

Implemented in RAMSES (final test phase)

See [Marchand et al. \(2021\)](#)

Download the **Ishinisan** code to calculate and tabulate the size-distributions.
<https://bitbucket.org/pmarchan/ishinisan>